

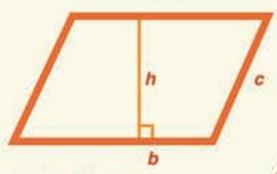


$$\frac{\pi}{4}$$

$$c = \sqrt{a^2 + b^2}$$

Perimeter

$$p = b + b + c + c$$
$$p = 2b + 2c$$



Area $A = bh$

$$A = \pi r^2$$



$$m(n) = O(n^4)$$

$$E_k = \frac{1}{2}mv^2$$

$$a^2 + b^2 = c^2$$

$$\Delta P = \rho g \Delta h$$

$$5 + 8 = 13$$
$$2 + 7 = 9$$
$$4 + 5 = 9$$
$$3 + 6 = 9$$
$$1 + 0 = 1$$

$$5 \times 2 = 10$$
$$4 \times 3 = 12$$
$$6 \times 7 = 42$$



FOR THE LOVE OF MATH

With the right lesson plan, teachers can turn struggling students into budding mathematicians. The secret is carefully guiding their adventure in numbers

BY JOHN MIGHTON

ILLUSTRATIONS BY CELIA JOHNSON

I still vividly remember the day, 14 years ago, when a tall and painfully shy sixth grade student named Lisa sat down at my kitchen table for her first math lesson with me. Lisa's principal had recommended her for a free after-school tutoring program I had started in my apartment with several friends. Although I had asked the principal for students struggling in math, I was not prepared for Lisa.

SAMPLE PROBLEM

“IF THE RATIO OF GIRLS TO BOYS IN A TOWN IS 4 TO 5, WHAT FRACTION ARE GIRLS?” MANY WILL ANSWER FOUR FIFTHS. THE CORRECT ANSWER IS FOUR NINTHS, WHICH REQUIRES UNDERSTANDING THAT THE DENOMINATOR OF THE FRACTION HAS TO REPRESENT ALL OF THE CHILDREN, NOT JUST THE BOYS.

I had planned to boost Lisa’s confidence by teaching her to add fractions. I knew from previous experience as a tutor that children often develop anxieties about math when they first encounter fractions. Because my lesson involved multiplication, I asked Lisa if she had trouble remembering any times tables, but she stared at me blankly. She had no idea what multiplication meant. Even the concept of counting by a number other than one was foreign to her. She was terrified by my questions

and kept saying, when I mentioned the simplest concepts, “I don’t understand.”

I had no idea what to do with Lisa, so I decided to see if she could learn to count by twos so she could eventually multiply by two. To ease her fears, I told her I was certain that she was smart enough to learn to multiply. I was afraid I might be giving her false praise, but my encouragement seemed to help her focus, and she made more progress than I expected.

I tutored Lisa once a week for three years. In grade nine, she transferred out of the remedial stream in math, and in her second term she skipped a year and enrolled in grade 10 math. She was able to solve word problems and carry out complex operations on tests independently, and several times I watched her teach herself material out of a textbook. Her final mark in grade 10 math was a

C+, but she was a year ahead. She had progressed from grade one to grade nine in only 100 hours of lessons, fewer than she would have received in a year of school. If I had had more time to prepare her, she could have done better.

In working with Lisa and the other students I tutored, I became convinced that children have far more potential to learn math than they typically exhibit at school. I registered my tutoring program as a charity, called JUMP (Junior Undiscovered Math Prodigies) Math, and embarked on a decade-long odyssey to determine the true math capabilities of children and why so many find the subject hard to learn. JUMP Math is now a classroom program used by more than 100,000 elementary and intermediate students in Canada. In the U.S., a number of school districts are testing a version of JUMP aligned with the Common Core State Standards.

From my experience as a tutor and then a volunteer in hundreds of classrooms, I developed a number of principles that form the basis of JUMP and are supported by research in educational psychology and cognitive science. They include providing lots of practice, giving students immediate feedback, teaching general math problem-solving strategies, and helping students discover new concepts by breaking down problems into small, manageable steps.

FAST FACTS

Formula for Success

1» Pure discovery-based math lessons can cause cognitive overload and therefore do not work as well as those in which a teacher helps a student to navigate a problem’s complexities.

2» Studies show that JUMP Math, a guided-discovery program the author developed, lifts students to much higher levels in math than most standard methods while shrinking the gap between weaker and stronger students.

3» The confidence that students gain by succeeding in math can bring broader benefits.

Surprisingly, these principles work not only in one-on-one lessons but also in whole classes. In a randomized controlled study presented at the Society for Research in Child Development in 2011, for example, cognitive scientists Tracy Solomon and Rosemary Tannock of the Hospital for Sick Children and the University of Toronto found that students from 18 classrooms using JUMP showed twice the rate of progress on a number of standardized math tests as those receiving standard instruction in 11 other classrooms. A large, multiyear pilot in inner-city schools in England, among many other anecdotal reports, have also shown that JUMP Math lifts students to much higher levels in math than most standard methods while dramatically shrinking the gap between weaker and stronger students.

Little Discovery

I believe that a root cause of many children's troubles in math, as well as in other subjects, is the belief in natural academic hierarchies. As early as kindergarten, children start to compare themselves with their peers and to identify some as talented or "smart" in various subjects. A child who decides that she is not talented will often stop paying attention or making an effort to do well. This problem will likely compound itself more quickly in math than in other subjects because when you miss a step in math it is usually impossible to understand what comes next. The more a child fails, the more her negative view of her abilities is reinforced and the less efficiently the child learns.

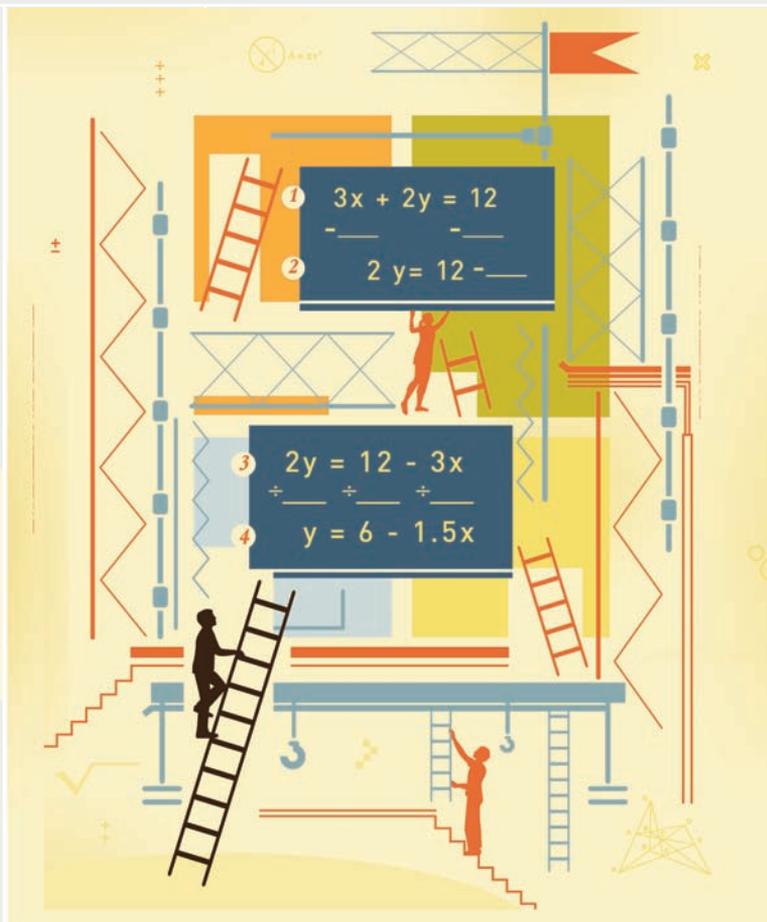
This belief in hierarchies causes greater differences between children in their success in math than do actual ability gaps. The fact that good instruction can dismantle hierarchies in math means that a child's current level of achievement need not dictate her long-term success in math.

In the past 15 years most schools in North America have adopted some kind of discovery- or inquiry-based math program, in which students are supposed to figure out concepts by themselves rather than being taught them explicitly. Discovery-based lessons tend to focus less on problems that can be solved by following a general rule, procedure or formula (such as "find the perimeter of a rectangle five meters long and four meters wide") and more on complex problems based on real-world examples that can be tackled in more than one way and have more than one solution ("using six square tiles, make a model of a patio that has the least pos-



sible perimeter"). Instead of memorizing facts and learning standard algorithms such as long division, students learn math primarily by exploring concepts and developing their own methods of calculation, mostly through hands-on activities with concrete materials.

Although I agree with many of the aims and methods of the discovery approach, a growing body of research suggests that some of its elements have significant drawbacks. To make math more relevant or appealing, for example, teachers will often select textbooks that are dense with illustrations or involve the use of concrete materials with engaging features, but these details can actually impede learning. For example, in 2013 psychologists Constance Kaminski and Vladimir Sloutsky of Ohio State University taught two groups of primary students to read bar graphs using two different types of graph: one had pictures of stacked shoes or flowers, and the other, more abstract graph had solid bars. Students who learned with the bars were better at reading graphs when the scale of the graph changed to reflect some multiple of the number of objects. Students taught with pictures tended to be distracted by counting the ob-



jects and so did not look at the scale on the graph.

Math teaching methods should not only avoid derailing students with extraneous material, they should also not saddle them with too much information at once. Discovery approaches can place a huge burden on working memory, the temporary mental scratchpad we use so heavily in solving math problems. Human working memory is extremely limited. On average, it holds the equivalent of about seven numbers at a time, a limit that the demands of a complex problem can easily exceed if the problem requires a lot of new knowledge.

Because of this hefty cognitive load, lessons based on pure discovery do not work as well as those in which a teacher helps a student navigate the complexities of a problem by providing feedback, working through examples and offering other guid-

(The Author)

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ance, according to a 2006 article by psychologist Paul Kirschner of the Open University of the Netherlands and his colleagues. The key is for a student to have mental capacity remaining to make inferences, integrate knowledge and reorganize information. “Empirical evidence collected over the past half-century consistently indicates that minimally guided instruction is less effective and less efficient than instructional approaches that place a strong emphasis on guidance of the learning process,” Kirschner and his co-authors wrote. In a 2011 meta-analysis (quantitative review) of 164 studies of discovery-based learning, psychologist Louis Alfieri of the City University of New York and his colleagues concluded: “Unassisted discovery does not benefit learners, whereas feedback, worked examples, scaffolding and elicited explanations do.”

Micro Inquiry

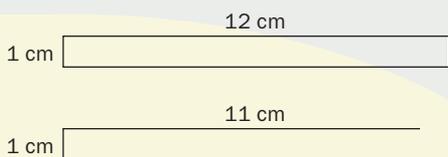
With these data in mind, we designed JUMP with supports such as additional examples, activities and practice. Based on a method called guided discovery or micro inquiry, JUMP lessons ask students to derive concepts and solve problems with a teacher’s guidance so that virtually all of the students succeed. What is more, the teacher’s direction closely aligns with certain principles that greatly assist the learning of mathematics.

One of these is scaffolding, which means breaking learning into chunks and providing relevant examples and practice to help students tackle each chunk. In a scaffolded lesson, concepts are introduced in a logical progression, with one idea leading naturally to the next. For instance, if you ask sixth graders this question, “If the ratio of girls to boys in a town is 4 to 5, what fraction are girls?” many will answer four fifths, or 80 percent. The correct answer is four ninths, which requires understanding that the denominator of the fraction has to represent all of the children, not just the boys.

In a scaffolded JUMP lesson on ratios and percentages, the first question asks students to write down the number of girls, boys and children in a class after statements such as “there are three girls and 10 children” and “there are six boys and five girls.” The next question asks the same thing, but students must also write down the fraction of girls and boys. Later, when students find the fraction of girls or boys in various problems, they must say if they are given the part and the whole, the two parts, or the “wrong” part and the whole. This sequence

of tasks stops students from forming a fraction by just putting one number over another without considering what the numbers mean. With this approach, it is possible to teach even the biggest ideas through a series of small steps.

Such lessons are designed to anticipate potential confusions or gaps in knowledge. To take a different example, if fifth grade students are asked to draw on graph paper all possible rectangles with whole-number sides that have a perimeter of 12 centimeters, many will start by drawing the figures shown below.



In the first instance, the student has confused perimeter with area; in the second, the child used up the allotted perimeter before completing the rectangle. To prepare students for the drawing exercise, the teacher might draw one side of a rectangle and allow students time to practice completing the drawing until they understand how the perimeter wraps around the figure. The teacher might also discuss a system for generating all the answers—say, by starting with a rectangle with width 1, then width 2, and so on.

Despite trying to prevent such confusions, students will still make plenty of mistakes, and those who repeat their errors will begin to doubt their abilities and lose hope that math can make sense to them. For this reason, JUMP Math lesson plans instruct teachers to provide immediate feedback and continuous assessment. During each JUMP lesson, which consists of a series of questions, exercises and challenges, the teacher selectively marks and discusses the student work as it is completed, so as to spot and correct errors and misconceptions before students move on, instead of testing the class a week later when it may be too late to help the ones who have fallen behind.

Another essential element of JUMP is to give kids general tools for solving math problems. Many adults struggle with the following elementary problem: “A person is standing 5,152nd in line, and a second person is 2,238th in line. How many people are between them?” Most people will subtract to find the answer, but if you ask them how they know

their answer is correct, they often will not be able to say. I know this approach will give the wrong answer, but not because I was born with an ability to see it. As a mathematician, I have learned basic strategies for solving problems, including this one: create an easier version of the problem and solve it instead. In this case, I would imagine five people in line and ask how many people are between the person who is fourth and the person who is second in

SAMPLE PROBLEM

“ONE PERSON IS STANDING 5,152ND IN LINE. ANOTHER IS 2,238TH. HOW MANY PEOPLE ARE BETWEEN THEM?” MOST PEOPLE SUBTRACT TO FIND THE ANSWER. BUT NOW IMAGINE A LINE OF FIVE PEOPLE AND ASK HOW MANY ARE BETWEEN THE SECOND AND THE FOURTH PERSON IN LINE. IS THE CORRECT ANSWER $4 - 2 = 2$?

line. Clearly, one person. From that simpler scenario, I immediately discover that subtracting the positions gives an answer that is one too high.

Practice Makes ...

I once tutored a student who had a severe attention deficit disorder and who had not managed to learn any multiplication facts by grade four. I told him I was going to give him a challenge: I would show him how to double large numbers mentally. I wrote:



I covered all but the millions part of the number with my hand and asked him to read what he could see. He said, “Two hundred thirty-four” and then “million.” I drew back my hand to reveal the thousand part, and he said, “One hundred twenty-two thousand.” When I exposed the rest of the number, he said, “One hundred forty-one.” As I had hoped, the boy was excited about reading this enormous number and asked to read more. My true goal,

SAMPLE PROBLEM

STUDENTS WHO CAN EASILY SEE THE RULE FOR THE SEQUENCE AT LEFT COULD BE GIVEN A BONUS QUESTION SUCH AS “SAY HOW MANY ODD NUMBERS IN THE SEQUENCE 1, 4, 7, 10 ... ARE LESS THAN 100.”

63
67
71
75

however, was to motivate him to multiply. So I then made a list of the first four entries of the two times table and showed him how to double a large number by doubling each digit and writing the result under the digit. While he was happily doubling numbers, the boy memorized the list and soon no longer needed it. He had practiced and learned part of the two times table in several minutes without being aware of it.

Although some educational theorists have made practice seem unnecessary or detrimental by calling it “drill and kill,” children need practice to become experts. The true challenge in education is thus to make practice interesting. If teachers make learning into a game with different levels and twists and turns as I did in my lesson with the big numbers, students will happily practice for a long time.

JUMP Math lesson plans also include extra “extension” questions that allow elementary and middle school teachers to give all students roughly the same lesson, without boring the stronger students or

holding them back. Ordinarily, teachers try to challenge faster students by giving them questions that introduce new concepts, skills and vocabulary. So the students need the teacher’s help, depriving the rest of the class of the teacher’s attention. In a JUMP lesson, an instructor assigns a set of incrementally harder bonus questions that kids can do on their own because they do not introduce new skills or concepts. Yet they feel as if they are surmounting a

series of interesting challenges, similar to what happens in a video game. For example, if a teacher asks a fourth grader to state a rule for the sequence “63, 67, 71, 75 ...” students for whom this answer is obvious could be given bonus questions such as “This sequence has a mistake in it; can you correct the mistake?” (Students love this.) Or “Find the missing numbers in the sequence 3, $_$, $_$, 18 ...” Or “Say how many odd numbers in the sequence 1, 4, 7, 10... are less than 100.”

Such questions help the more advanced students gain a deeper understanding of the curriculum. In addition, when youngsters who are initially slower see that they can handle the work, they speed up so they can get bonus questions, too. Indeed, teachers who follow the JUMP lesson plans with fidelity can

dramatically close the gap between weaker and stronger students.

For example, in the fall of 2007 fifth grade teacher Mary Jane Moreau of the Mabin School in Toronto gave her students a standardized assessment called the Test of Mathematical Abilities. The class average was in the 54th percentile, with a wide range of scores, including one student who ranked at just the ninth percentile. A fifth of the pupils were identified as learning disabled. Moreau then abandoned her usual approach, which meant pulling together lessons with the best materials she could find, and followed the JUMP lesson plans. After a year of JUMP, the class average rose to the 98th percentile, with the lowest mark in the 95th percentile. After two years of JUMP, 17 of her 18 students signed up for the Pythagoras Math competition, a prestigious contest for sixth graders, and 14 of them received awards of distinction (with the other three close behind).

This case is not isolated. In 2006 Nikki Aduba, a math consultant for Lambeth, one of the neediest

boroughs in England, enlisted teachers to use JUMP with 159 students who were a year below grade level at the beginning of grade six. Almost all the students had learning disabilities or behavior problems, and few were expected to pass the national exams in math. A year later 69 percent of them had advanced about two years in math, and 60 percent passed the exams. Aduba reported similar results with hundreds of students in various grades four years in a row. In another case, Muheim Elementary School in northern British Columbia for years ranked in the bottom 10 percent in its school district on provincial math tests. Since the principal introduced JUMP Math five years ago, the school has held its position among the top 10 percent.

Confidence Boost

None of the basic learning strategies used in JUMP are radical or even new in education. But JUMP has applied them with rigor, paying close attention to the order and size of steps, the amount of review provided and the methods of questioning, among other details. If there is anything new about JUMP, it lies in the assumptions that guided its development, including the idea that almost all students can achieve more in math than schools require. JUMP assumes that children who believe in their abilities can enjoy doing math as much as they enjoy making art or playing sports. It is fun to overcome challenges and exercise the mind, and it can be thrilling to discover or understand something that is beautiful, useful or new.

The confidence that students gain by succeeding in math can have effects in other parts of their lives. Because math is supposed to be hard, when children think they are capable of learning math they tend to think that they can learn anything. In Lambeth, one teacher reported that students with behavior problems would reprimand others who misbehaved in math class because they were so engaged in their lessons. Another teacher wrote that her students had become “ballsy, independent problem solvers.” I once taught 11-year-olds in Lambeth how to read binary codes, the strings of 0s and 1s that represent numbers for computers. The students seemed to think they were little code breakers and demanded longer and longer codes. On my third day at the school, when the teacher and I entered the classroom the children cheered.

Children love solving puzzles, seeing patterns and making connections. They have a sense of won-



der that is diminished only by failure. In the past decade cognitive scientists and educators have begun to uncover the mechanisms by which our brains learn best, and they have gathered evidence that the significant majority of students can excel at and love learning subjects such as math. One of the most important questions of our time is whether we will act on that evidence by educating children according to their extraordinary potential. **M**

(Further Reading)

- ◆ **Applications and Misapplications of Cognitive Psychology to Mathematics Instruction.** John R. Anderson, Lynne M. Reder and Herbert A. Simon in *Texas Education Review*, Vol. 1, No. 2, pages 29–49; Summer 2000.
- ◆ **The Expert Mind.** Philip E. Ross in *Scientific American*, Vol. 295, No. 2, pages 64–71; August 2006.
- ◆ **The End of Ignorance: Multiplying Our Human Potential.** John Mighton. Vintage Canada, 2008.
- ◆ **Does Discovery-Based Instruction Enhance Learning?** Louis Alfieri et al. in *Journal of Educational Psychology*, Vol. 103, No. 1, pages 1–18; February 2011.
- ◆ To download JUMP Math lesson plans for grades one to eight, teachers and parents can go to jumpmath.org